How We Combine Factors Matters!

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For nearly 40 years, academic researchers and quantitative equity managers have been in an endless quest to find factors that predict stock returns. Harvey, Liu, and Zhu (2016), document 314 factors – albeit many of them highly correlated – that have been identified in the finance academic literature. Yet very little work has been done on the best way to combine the plethora of factors to predict stock returns in an *efficient manner*.[1] What is the best way to put some structure on the factor dimensionality and "tame the factor zoo?" Obviously, for practitioners the emphasis should be on robust *out-of-sample* predictions rather than explaining stock returns *in-sample*. We believe that to achieve this, stock selection models should be parsimonious and have only a few key parameters to estimate. Over-engineered and data-mined models may show great results in-sample but are unlikely to produce good results out-of-sample, due to the estimation errors caused by randomness in data. The combined predictive ability of the factors will be enhanced when there is a shrinkage of their dimensionality because of the reduction in estimation error and noise.

In this paper, we will demonstrate that 'how to combine factors' matters a lot. There are two key points that arise from our back-test results. First, stock selection models that shrink the dimensionality and use fewer parameters are more reliable and solid over time. Secondly, this dimension reduction is best achieved when structure is placed on the factors by relying on *both* economic intuition and statistical evidence.

COMPOSITE FACTORS

We classify 69 underlying factors into nine factor groups, where each group of factors measures some economic concept or characteristic of the stocks, such as valuation, profitability, momentum, capital-use, and risk. Within each factor group, apart from the economic meaning of the factors being similar, we ensure that the cross-sectional correlation among the them is high. We combine the information from the factors within a group into a composite factor by averaging their percentile ranks and re-ranking the composite factor. (We use percentile ranks to mitigate the effects of outliers.) We thus harness the information from each factor and yet reduce noise by averaging the factors. Note that towards our goal of having only a few parameters, our averaging process gives equal weight to all factors within a composite group. This avoids data mining and attempting to pick the "best" factor within each factor cluster or pruning down the potential factor list based purely on recent past performance.

[1] In the academia, only recently there has been a recognition of the importance of how to combine the various factors that predict stock returns. See for example, Feng, Giglio, Xiu (2017), Green, Hand, Zhang (2014), Han, He, Rapach, Zhou (2018), Huang, Li, Zhou (2018), Kelly, Pruitt, Su (2018), Kozak, Nagel, Santosh (2017), and Light, Maslov, Rytchkov (2018).

We choose nine factor composites with the objective of having parsimonious factor groupings that can capture the various stock characteristics. Each of these composites captures a distinct stock characteristic intended to exploit a behavioral bias among investors. Thus, the cross-sectional correlation between the factor composites is low to moderate, while each of them has an ability to predict stock returns in the cross-section. In our approach, the weight assigned to each factor composite is based on the magnitude and consistency of the factor premiums, estimated in multi-variate cross-sectional Fama-Macbeth (FMB) regressions. Consequently, the weights on some of the factors, such as value and momentum, are higher than the others. Also, the nine-factor structure gives us ability to vary their weights dynamically through time depending on the prevailing macro and market conditions.

HORSE RACE

We now demonstrate the strength of our modeling approach by comparing its back-test performance to other potential ways to combine the 69 component factors. Specifically, we examine four other approaches identified in the finance literature. We test the ability of each approach to predict the average excess returns of quintile groups of stocks in our custom Emerging Markets universe of approximately 1,000 stocks over the January 2000 - March 2018 period.

In our modeling approach, the nine composite weights reflect the average multi-variate factor premiums and their variability through time. Starting in December 1999, we determine the weight for each factor composite based on the factor-premium history available from January 1995 until the end of each month, in an expanding-window. We then create a model score for each stock as a weighted score of the nine factor-composite ranks. Next, based on these model scores, we rank the stocks each month, and measure the out-of-sample stock return performance in the following month. We repeat this each month in an expanding window to create the back-test history from January 2000 to March 2018. Chart 1, data set (1), shows the statistics for the excess returns relative to universe average of equally-weighted quintile portfolios based on the model ranks. The average annualized excess return (“ER”) for the top quintile is 9.3% with an information ratio (“IR”) of 1.47. The return spread between the extreme quintiles is 19%.

As a first contender to compare our methodology, we examine a model that uses all the underlying 69 component factors directly in a multi-variate Fama-Macbeth (FMB) methodology, as in Green, Hand, Zhang (2014). Specifically, each month, starting in January 1995, we estimate a cross-sectional regression of stock returns on the percentile ranks ($F_{i,t}$) of the 69 factors at the end of the previous month.

$$R_{i,t} = \gamma_{0,t} + \gamma_{1,t}F^{1}_{i,t-1} + \gamma_{2,t}F^{2}_{i,t-1} + \ldots + \gamma_{K,t}F^{K}_{i,t-1} + \epsilon_{i,t}$$

[2] We use an expanding-window approach to simulate the real-world experience of investors by relying only on data and information that would have been available at each point in time

We then estimate the expected factor-premiums each month, in an expanding-window, by averaging the realized monthly factor-premiums until then.

$$E(\gamma_{k,T}) = \sum_{t=1}^{T} \gamma_{k,t}/T$$

Given the expected factor-premiums and the 69 factor-ranks, we generate the expected stock returns as follows:

$$E(R_{i,t}) = \sum_{k=1}^{K} E(\gamma_{k,t}) F_{i,t}^{k}$$

We must point out at least two problems that we encounter in employing this methodology. First, many of these factors are highly correlated so the estimated factor-premiums are very unstable because of multi-collinearity. Secondly, the multi-variate regression specification requires that we have data for all the 69 factors for the stock to be included in the regression. In other words, a stock can get dropped from the analysis even if one of the factors is missing data. This especially is an issue in the earlier years of the back-tests. We mitigate this problem by assuming a neutral factor rank of 50 whenever a stock has a missing factor rank. Note that this problem does not arise in our composite factor rank approach since we average the factor ranks within each factor cluster using the data available for a stock. In any case, we generate stock ranks each month for the 1,000 or so EM stocks based on the expected stock returns using the 69 factor ranks. Chart 1, data set (2), shows the performance of the portfolios formed by these stock ranks. As compared to our approach, the average performance of the quintile portfolios is much weaker. Overall, the top quintile only has an average excess return of 1.9%. This illustrates the weakness of an approach that is over-specified with 69 factor premiums to be estimated!

Second, we look at the Partial Least Square (*PLS*) regression approach suggested by Light, Maslov, Rytchkov (2018). This methodology attempts to simplify the estimation problem by using only the univariate factor premiums, where each factor's ability to predict stock returns is estimated individually. Specifically, we first estimate a cross-sectional regression of $R_{i,t}$ on each individual factor rank, $F^{k}_{i,t-1}$, for the 69 factors, *k*=1, ...,K, and obtain the univariate factor premiums λ_t^k. Just as in the methodology above, we estimate the expected factor premium at time *t*, by averaging the realized factor premiums in an expanding-window through time, $\overline{\lambda_t^k}$. Next, for each stock *i*, and month *t*, we generate a composite stock signal, $S_{i,t}$, by regressing the 69 factor ranks, $F_{i,t}^k$, on their estimated factor premiums $\overline{\lambda_t^k}$. Now, we can rank the stocks each month using this composite signal and see their predictive ability of returns in the following month. As shown in Chart 1, data set (3), the average annualized excess return of 5.8% (with IR of 0.92) for the top quintile portfolio is much better than the earlier approach that estimates all the 69 factor premiums simultaneously. So, the factor dimensional reduction using the PLS model does help, but it is still not as robust as our approach of using nine composite factors.

The third approach we examine is estimating the expected stock returns by averaging the forecasts from each of the 69 univariate factor premiums. That is, the expected stock return for stock *i,* at time *t*, is $\sum_{k=1}^{K}(\lambda_t^k\, F_{i,t}^k)/K.$ We then rank the stocks based on these forecasted stock returns. This is clearly a simpler approach than the PLS method where the composite signal puts greater weight on factors with larger average factor premiums. This method puts equal weight on the forecasts from each of the 69 factors. Interestingly, this simple average forecast method is more robust out-of-sample. The top quintile portfolio has average excess return of 8.3% (with IR of 1.27), shown in Chart 1, data set (4).

Finally, we look at an even simpler approach where the overall stock rank is based on an average of the 69 factor ranks, $F_{i,t}^k$. Remarkably, this approach, which does not involve estimating any factor premiums, has the best predictive ability for the cross-section of stock returns. The average excess return of 8.5% for the top quintile (see Chart 1, data set (5)) is the best among all the four alternatives we considered. Notably, our model, that reduces the factor dimensionality problem by classifying the 69 factors into nine composite factors based on economic logic, does the best overall and dominates all the alternatives. Also, recall just as in the last approach, we generate the composite factor rank by averaging the factor ranks within a given group.

CONCLUSION

There are some strong insights gained from our out-of-sample back-tests over nearly 18 years. First, the most robust out-of-sample results are obtained from the simplest method with little parameter estimation. Second, placing equal weight to the factors irrespective of their in-sample performance generates better forecasts. This also implies that trying to pick the “best” factor within a factor group will be futile. Third, reducing the factor dimensionality by relying on the economic import of each factor is better than just using statistical techniques. Also, it has the additional benefit of permitting us to vary the weights dynamically among the nine factor composites depending on the prevailing market and macro-economic context. In summary, we believe our stock selection model offers a reliable way to rank and pick stocks, by harnessing the information in the 69 factors efficiently. How we combine factors matters!

Chart 1

Back-Tests of Methodologies to Combine Factor Ranks into Overall Stock Ranks
in Custom EM Universe, January 2000 – March 2018

	(1) FMB Model of Composite Factors			(2) Multivariate FMB Model of Individual Factors			(3) PLS Model of Univariate Factor-Premiums			(4) Average Forecasts from Univariate FMB Regressions			(5) Average Factor Rank		
Quintile	ER	IR	Skew	ER	IR	Skew	ER	IR	Skew	ER	IR	Skew	ER	IR	Skew
Top	9.3	1.47	0.10	1.9	0.38	1.44	5.8	0.92	-0.50	8.3	1.27	0.14	8.5	1.49	0.24
4	3.2	0.83	-1.18	0.9	0.27	-0.46	3.1	0.66	-1.53	2.8	0.76	-0.79	2.4	0.63	-1.24
3	0.3	0.09	-0.03	-1.1	-0.35	-0.13	-0.5	-0.16	-0.30	0.5	0.15	-0.16	-0.1	-0.03	-0.05
2	-3.1	-0.81	-0.43	-2.2	-0.64	-0.29	-1.7	-0.42	0.18	-2.6	-0.63	0.37	-2.4	-0.72	0.22
Bottom	-9.7	-1.47	0.97	0.4	0.08	0.20	-6.7	-0.81	1.49	-9.1	-1.37	1.55	-8.4	-1.24	0.95

Source: Chicago Equity Partners, LLC

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